Solving 3x3 Determinant using 2-column method

Example $\begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$

Step 1: Copy column 1 and 2 next to the matrix.

$\begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} 2 & 1 \\ 3 & -1 \\ 1 & 1 \end{matrix}$

Step 2: Beginning with upper left hand corner, multiply the numbers on the diagonal (3 numbers only).

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} 2 & 1 \\ 3 & -1 \\ 1 & 1 \end{matrix}$$

To that add the product of the 3 numbers on the next diagonal.

And again, add the product of the 3 numbers on the last diagonal.

$$2(-1)2 + 1(4)(1) + 0(3)(1) = -4 + 4 + 0 = 0$$

Now beginning with the 1 in the upper right hand corner, we are going to come back, multiplying the numbers on the diagonals. We will also sum these and then subtract the answer from the sum above.

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} 2 & 1 \\ 3 & -1 \\ 1 & 1 \end{matrix}$$

$$1(3)(2) + 2(4)(1) + 0(-1)(1) = 6 + 8 + 0 = 14$$

Now subtract: 0 - 14 = - 14

Now, let's use this new skill to solve a 3x3 by Cramer's Rule

This is the first problem on your handout for tonight. We will do this TOGETHER.

(the use of determinants to solve)

$$-2x + y - z = 2$$
$$-x - 3y + z = -10$$
$$3x + 0y + 6z = -24$$

$$D = \begin{vmatrix} -2 & 1 & -1 \\ -1 & -3 & 1 \\ 3 & 0 & 6 \end{vmatrix} \begin{matrix} -2 & 1 \\ -1 & -3 \\ 3 & 0 \end{matrix} =$$

$$(-2)(-3)(6) + (1)(1)(3) + (-1)(-1)(0)$$
$$36 + 3 + 0 = 39$$

$$(1)(-1)(6) + (-2)(1)(0) + (-1)(-3)(3)$$
$$-6 + 0 + 9 = 3$$

$$39 - 3 = \boxed{36}$$

$$D_x = \begin{vmatrix} 2 & 1 & -1 \\ -10 & -3 & 1 \\ -24 & 0 & 6 \end{vmatrix} \begin{matrix} 2 & 1 \\ -10 & -3 \\ -24 & 0 \end{matrix}$$

$$= 2(-3)(6) + (1)(1)(-24) + (-1)(-10)(0)$$
$$-36 - 24 + 0 = -60$$

$$1(-10)(6) + 2(1)(0) + (-1)(-3)(-24)$$
$$-60 + 0 - 72 = -132$$

$$-60 - (-132) = \boxed{72}$$

$$D_y = \begin{vmatrix} -2 & 2 & -1 \\ -1 & -10 & 1 \\ 3 & -24 & 6 \end{vmatrix} \begin{matrix} -2 & 2 \\ -1 & -10 \\ 3 & -24 \end{matrix}$$

$$-2(-10)(6) + 2(1)(3) + (-1)(-1)(-24)$$
$$120 + 6 - 24 = 102$$

$$2(-1)(6) + (-2)(1)(-24) + (-1)(-10)(3)$$
$$-12 + 48 + 30 = 66$$

$$102 - 66 = \boxed{36}$$

$$D_z = \begin{vmatrix} -2 & 1 & 2 \\ -1 & -3 & -10 \\ 3 & 0 & -24 \end{vmatrix} \begin{matrix} -2 & 1 \\ -1 & -3 \\ 3 & 0 \end{matrix} =$$

$$-2(-3)(-24) + (1)(-10)(3) + 2(-1)(0)$$
$$-144 - 30 + 0 = -174$$

$$(1)(-1)(-24) + (-2)(-10)(0) + 2(-3)(3)$$
$$24 + 0 - 18 = 6$$

$$-174 - 6 = -180$$

$$X = \frac{D_x}{D} = \frac{72}{36} = 2$$

$$y = \frac{D_y}{D} = \frac{36}{36} = 1$$

$$z = \frac{D_z}{D} = \frac{-180}{36} = -5$$

$$\boxed{(2, 1, -5)}$$

$$\boxed{\text{check all 3}}$$